Semantic and Declarative Technologies

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Course information

- Course layout
  - Introduction to Logic   Weeks 1–2
  - Declarative Programming
    - Prolog – Programming in Logic   Weeks 3–7
    - Constraint Programming   Weeks 8–12
  - Semantic Technologies
    - Logics for the Semantic Web   Weeks 13–14
- Requirements
  - 2 assignments (150 points each)   300 points
  - 2 tests (mid-term and final, 200 points each)   400 points total
  - many small exercises + class activity   300 points total
- Course webpage: `https://ait-sdt.hu/`
- Course rules: `https://ait-sdt.hu/course-rules.pdf`

Part I: Declarative Programming – the Prolog language

- Prolog – PROgramming in LOGic
  - The program = statements in simplified first-order logic (FOL)
  - Program execution is a very simple reasoning process explainable as pattern-based procedure invocation with backtracking
- Dual semantics: declarative and procedural
  - Slogan: WHAT rather than HOW
    (focus on the logic first, but then think over Prolog execution, too).

Part II: Declarative Programming with Constraints

- The CLP($\mathcal{X}$) schema

Prolog or some other programming language, e.g. C++	+	“strong” reasoning capabilities on a restricted domain $\mathcal{X}$ involving specific constraint (relation) and function symbols.

- Examples for the domain $\mathcal{X}$:
  - $\mathcal{X} =$ Q or R (rationals or reals)
    constraints: linear equalities and inequalities
    reasoning techniques: Gauss elimination and the simplex method
  - $\mathcal{X} =$ B (Boole values `true` and `false`, or `1` and `0`)
    constraints: propositional operators ($\wedge, \vee, \neg$, etc.)
    reasoning techniques: SAT solvers, using e.g. Binary Decision Diagrams.
  - $\mathcal{X} =$ FD (Finite Domains, e.g. of integers)
    constraints: arithmetic, logic, and combinatorial relationships
    reasoning techniques: those developed for Constraint Satisfaction Problems (CSPs)
- Main topic: Finite domain constraints (base constraints, reification, combinatorial and user defined constraints, disjunctions, modelling)

Part III: The Semantic Web

- The goal: making the information on the web processable by computers
- Slogan: machines should be able to understand the web, not only read it
- To achieve the vision of the Semantic Web one has to:
  - add formal meta-information to web pages (ABox), e.g.
    (AIT hasLocation Budapest) (AIT hasType University)
    (Paul hasParent Peter) (Paul hasType Parent)
  - formalise background knowledge – build terminologies (TBox)
    - hierarchies of notions, e.g.
      a Father is a (subconcept of) Parent,
      the hasFather relationship is contained in the hasParent relation
    - definitions and axioms, e.g.
      a Father is a Male Person having at least one child (i.e. having at least one object in hasChild relationship).
  - develop reasoning algorithms and tools
- Main topics: Description Logics, from $\mathcal{AL}$ to $\mathcal{SROIQ}(\mathbf{D})$, reasoning tasks, ABoxes; a brief overview of reasoning algorithms; OWL (Web Ontology Language) – the language of the Semantic Web

Part I – *practical* mathematical logic

Propositional Logic

- Basic Boolean functions (bitwise ops in C, Python, etc.)
  - `and`: $\wedge$ (&)
  - `or`: $\vee$ (|)
  - `not`: $\neg$ (~)
  - `implies`: $\rightarrow$ $A \rightarrow B$ (A implies B) is the same as $(\neg A \vee B)$
- The puzzle below is cited from “What Is The Name Of This Book?” by Raymond M. Smullyan, chapter “From the cases of Inspector Craig”
- Puzzles in this chapter involve suspects of a crime, named A, B, etc. Some of them are guilty, some innocent.
- Example:
  An enormous amount of loot had been stolen from a store. The criminal (or criminals) took the heist away in a car. Three well-known criminals A, B, C were brought to Scotland Yard for questioning. The following facts were ascertained:
  1. No one other than A, B, C was involved in the robbery.
  2. C never works without A (and possibly others) as an accomplice.
  3. B does not know how to drive.

  Is A innocent or guilty? Can you answer the same question for B or C?

Inspector Craig puzzle – transforming to formal logic

- Let's recall the facts
  1. No one other than A, B, C was involved in the robbery.
  2. C never works without A (and possibly others) as an accomplice.
  3. B does not know how to drive.
- Transform each statement into a formula involving the letters A, B, C as atomic propositions. Proposition A stands for "A is guilty", etc.
  1. A is guilty or B is guilty or C is guilty: $A \vee B \vee C$
  2. If C is guilty then A is guilty: $C \rightarrow A$
  3. It cannot be the case that only B is guilty: $B \rightarrow (A \vee C)$
- Transform each propositional formula into conjunctive normal form (CNF), then show the clauses in simplified form:

	Original formula	CNF	Simplified clausal form
1	$A \vee B \vee C$	$A \vee B \vee C$	`+A +B +C.`
2	$C \rightarrow A$	$\neg C \vee A$	`-C +A.`
3	$B \rightarrow (A \vee C)$	$\neg B \vee A \vee C$	`-B +A +C.`

- A clause is a set of signed atomic propositions, called *literal*s

Inspector Craig puzzle – resolution proof

- Collect the clauses, giving each a reference number:

(1)	+A +B +C.	Only *A*, *B*, *C* was involved in the robbery.
(2)	-C +A.	*C* never works without *A* as an accomplice.
(3)	-B +A +C.	*B* does not know how to drive.

- A resolution step requires two input clauses which have opposite literals e.g. literal 3 of clause `(1)` is `+C` while lit 1 of clause `(2)` is `-C`
- The resolution step creates a new clause, called the resolvent. It takes the union of the literals in the inputs and removes a single pair of opposite literals, e.g. resolving (1) lit 3 with (2) lit 1 results in `+A +B`
- The resolvent follows from (is a consequence of) the input clauses, as $(U \vee V) \wedge (\neg U \vee W) \rightarrow (V \vee W)$ always holds (is a tautology)
- A sample resolution proof:

		resolve (1) lit 2 with (3) lit 1 resulting in (4)
(4)	+A +C.	resolve (4) lit 2 with (2) lit 1 resulting in (5)
(5)	+A.	

- We deduced that *A* is true, so the solution of the puzzle is: *A* is guilty

Clauses in First Order Logic (FOL)

- Example: There is an island where some people are optimistic (opt)
- Given the facts below, can you deduce that someone is optimistic?
  1. Those having an opt parent are bound to be opt.
  2. Those having a non-opt friend are also bound to be opt.
  3. Susan's mother has Susan's father as a friend.
- To formalize this in FOL we introduce some task-specific symbols:
  - X has a parent $Y \longrightarrow \texttt{hasP}(X, Y)$; X has a friend $Y \longrightarrow \texttt{hasF}(X, Y)$
  - X is opt $\longrightarrow \texttt{opt}(X)$; $\texttt{s}$, $\texttt{f}$, $\texttt{m}$ stand for Susan, her father and her mother, resp.
- The FOL form and the clausal form of the above statements:
  1. For all X and Y, X is opt if X has a parent Y and Y is opt:
     $\forall X, Y.(\texttt{opt}(X) \leftarrow \texttt{hasP}(X, Y) \land \texttt{opt}(Y))$
     `+opt(X) -hasP(X,Y) -opt(Y).`
  2. For all X and Y, X is opt if X has a friend Y and Y is not opt:
     $\forall X, Y.(\texttt{opt}(X) \leftarrow \texttt{hasF}(X, Y) \land \neg\texttt{opt}(Y))$
     `+opt(X) -hasF(X,Y) +opt(Y).`
  3. $\texttt{hasP}(\texttt{s}, \texttt{m})$ $\texttt{hasP}(\texttt{s}, \texttt{f})$ $\texttt{hasF}(\texttt{m}, \texttt{f})$
     `+hasP(s,m). +hasP(s,f). +hasF(m,f).`
- You will learn a reasoning algorithm FOL resolution, capable of finding an optimistic person. Prolog execution is based on FOL resolution.

Part II – Prolog

Example 1: checking if an integer is a prime

- A Prolog program consists of predicates (functions returning a Boolean)
- Let’s write a predicate, which is true if and only if the argument is a prime
- Programming by specification: first describe when the predicate is true, then transform the decription to Prolog code

```
prime(P) :-                 % P is a prime if
  integer(P), P > 1,        %     P is an integer and P > 1 and
  P1 is P-1,                %     P1 = P-1 and
  \+ (                      %     it is not the case that
                            %     (there exists an integer I such that)
    between(2, P1, I),      %         2 =< I =< P1 and
    P mod I =:= 0           %         P is divisible by I
  ).                        %
```

Are you convinced of the correctness of the code? `:-)`

Example 2: append - multiple uses of a single predicate

- `app(L1, L2, L3)` is true if `L3` is the concatenation of lists `L1` and `L2`.

```
app([], L, L).                 % appending an empty list with L gives L.
app([H|L1], L2, [H|L3]) :-     % appending a list composed of
                               % head H and tail L1 with a list L2
                               % gives a list with head H and tail L3 if
    app(L1, L2, L3).           %     appending L1 and L2 gives L3.
```

- app can be used, for example,
  - to check whether the relation holds:

    `| ?- app([1,2], [3], [1,2,3]).` $\Longrightarrow$ `yes`
  - to append two lists:

    `| ?- app([1,2], [3,4], L).` $\Longrightarrow$ `L = [1,2,3,4] ? ; no`
  - to split a list into two:

```
| ?- app(L1, L2, [1,2,3]).     ==>  L1 = [], L2 = [1,2,3] ? ;
                                    L1 = [1],  L2 = [2,3] ? ;
                                    L1 = [1,2],  L2 = [3] ? ;
                                    L1 = [1,2,3], L2 = [] ? ; no
```

- Predicate `app` is available as a built-in: `append/3` (`append` with 3 args)

Example 3: A number puzzle

- An arithmetic expression is simple if it uses the four basic operations only
- Let's write a Prolog program for solving the following task:
  Given a set of integers, e.g. $\{1, 3, 4, 6\}$, and a target integer *n*, e.g. 14, build a simple arithmetic expression that contains each element of the given set exactly once, and evaluates to *n*
- Some further clarification:
  - you cannot "glue" together integers to form larger ones, e.g. forming 13 from 1 and 3 is not allowed
  - each operation can be used 0 or more times
  - parentheses can be used freely
- Examples: $1 + 6 * (3 + 4) = 43$, $(1 + 3)/4 - 6 = -5$
- The list of integers contained within an expression (in order of occurence) is called its list of leaves, e.g. the list of leaves of $6 * (3 + 4)$ is `[6,3,4]`
- A fairly hard task is to construct an expression that evaluates to 24, using integers $\{1, 3, 4, 6\}$

The number puzzle in Prolog

Blue/orange color indicates built-in/library predicates

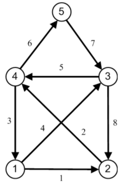
```
% Expr uses all integers in L and evaluates to Val.
leaves_value_expr(L, Val, Expr) :-
    permutation(L, PL),      % PL is a permutation of L,
    leaves_expr(PL, Expr),   % PL is a list of leaves of Expr,
    catch(Expr =:= Val, _,   % Expr evaluates to Val, if any error
          fail).             % occurs (e.g. division by 0), simply fail

% Expr is an (arbitrary) expression having a given list of leaves L.
leaves_expr(L, Expr) :-
    L = [Expr].              % If L is a singleton, Expr is the element
leaves_expr(L, Expr) :-
    append(L1, L2, L),       % Split L into L1 ⊕ L2
    L1 \= [], L2 \= [],      % so that neither L1, nor L2 is empty ([])
    leaves_expr(L1, E1),     % Let E1 be an arbitrary expr with leaves L1
    leaves_expr(L2, E2),     % Let E2 be an arbitrary expr with leaves L2
    member(Op, [+,-,*,/]),   % Let Op be one of the four allowed operations
    Expr =.. [Op,E1,E2].     % Let Expr be a binary expresion
                             % with operation Op and operands E1 and E2
```

Example 4: Finding an Euler walk in a graph

- From Wikipedia (`https://en.wikipedia.org/wiki/Eulerian_path`):

  An Eulerian trail, or Euler walk, in an undirected graph is a walk that uses each edge exactly once.
  Euler's conjecture: *For the existence of an Euler walk it is necessary that zero or two vertices have an odd degree.*
  This holds for the "House of Santa Claus" graph on the right: only vertices 1 and 2 have odd degrees.



- An edge of a graph between `A` and `B` is represented by a Prolog term `A-B`
- A graph is represented by a list of edges, e.g.
  `[1-2,1-3,1-4,2-3,2-4,3-4,3-5,4-5]` represents the graph above
- Another representation is `[1-2,2-4,4-1,1-3,3-4,4-5,5-3,3-2]`, which corresponds to the above Euler walk (see the numbers along the edges)
- Let's count the number of representations of a graph with n ($= 8$) edges:
  - each edge can be written in two ways, giving 2^n (256) arrangements
  - the n edges can be arranged in $n!$ ($8! = 40320$) ways
  - the number of representations is $2^n \cdot n!$, for $n = 8$ this is $\sim$ 10 million

Euler walks – a simple solution

```
% draw0(G, W): W is a walk representing the same graph as G.
draw0(G0, W) :-           % G0 is the same graph as the walk W if
   same_graph0(G0, W),    % G0 and W represent the same graph and
   walk(W).               % W is a walk

% walk(G): graph G is an Euler walk.
walk([_-_]).              % A graph consisting of a single edge is a walk.
walk(G) :-                % A graph G is a walk if
   G = [_-Q|T],           %    the endpoint Q of its first edge is the same
   T = [Q-_|_],           %    as the starting point of the second edge, and
   walk(T).               %    the tail of G, T, is a walk.

% same_edge(E0, E): E0 and E represent the same (undirected) edge
same_edge(A-B, A-B).                 same_edge(A-B, B-A).

% same_graph0(G0, G): G0 and G represent the same graph
same_graph0(G0, G) :-     % G0 and G represent the same graph if
   maplist(same_edge,     % G1 is obtained from G0 by reversing some
           G0, G1),       % (0 or more) edges, and
   permutation(G1, G).    % G is obtained from G1 by permuting the edges
                          % (permutation/2 is defined in library(lists))
```

- `draw0` finds all solutions in 2.7s (run on a processor i5-6300U @ 2.40GHz)

Euler walks – merging edge reversal and permutation

- Permutation relies on another library predicate: `select/3`:

```
% select(E, L, RestL): E occurs in list L, and list RestL is obtained
%                      by removing this occurrence of E from L.
%                      Concisely: selecting E from L leaves Rest
```

- This is one of the ways to implement permutation:

```
% permutation(L, P): list L has a permutation P.
permutation([], []).           % [] has a permutation [].
permutation(L, [First|Perm]):- % L has a permutation [First|Perm] if
    select(First, L, Rest),    %   selecting First from L leaves Rest,
    permutation(Rest, Perm).   %   Rest has a permutation Perm.
```

- Inserting edge reversal into the above code yields this predicate:

```
% same_graph1(G0, G): G0 and G represent the same graph
same_graph1([], []).
same_graph1(L, [E|Perm]) :-  % L is the same graph as [E|Perm] if
    select(First, L, Rest),  %    selecting First from L leaves Rest,
    same_edge(First, E),     %    First represents the same edge as E,
    same_graph1(Rest, Perm). %    Rest is the same graph as Perm.

draw1(G0, W) :- same_graph1(G0, W), walk(W). % All solutions in 3.5s :-(.
```

Euler walks – breakthrough

- To make the code faster we reorder subgoals and add a redundant goal:

```
draw1(G0, W) :-                   draw2(G0, W) :-
    /* generate-and-test */            same_length(G0, W),   % G0 and W are of
                                       walk(W),              % equal length
    same_graph1(G0, W),                same_graph1(G0, W).
    walk(W).                           /* constrain-and-generate */
```

- `draw2` completes in 0.43 msec, 4 magnitudes faster than `draw1/draw0`
- `same_length/2` is a library predicate from `library(lists)`. It ensures that the two arguments are lists of the same length:

```
| ?- same_length([1,2,3], L).     ⟹     L = [_A,_B,_C] ? ; no
```

- If one of the arguments is a variable (as `L` above), it will be instantiated to a list of appropriate length, containing distinct variables
- What happens when `walk/1` is called with such an argument?

```
| ?- G = [_,_,_], walk(G).        ⟹     G = [_A-_B,_B-_C,_C-_D] ?
```

- Variable `G` obtains a “walk pattern”: the endpoint of the first edge (`_B`) is the same variable as the starting point of the second edge, etc. Whenever one of these variable occurrences gets instantiated, the other occurrence gets the same value, reducing the search space considerably.

Prolog extensions: coroutining (Prolog II)

- Wikipedia: Coroutines are computer program components that allow execution to be suspended and resumed, generalizing subroutines for cooperative multitasking. Coroutines are well-suited for implementing familiar program components such as cooperative tasks, exceptions, event loops, iterators, infinite lists and pipes.
- A typical example of coroutining, the Hamming problem: Generate, in increasing order, the sequence of all positive integers divisible by no primes other than 2, 3, 5.
- We implement a simplified version: the only divisors allowed are 2 and 3, using predicates `times/3` and `merge/3` in dataflow programming style
- Merge two sorted lists into a single sorted list

```
% merge(As, Bs, Cs): Sorted list Cs is obtained by
% collating sorted lists As and Bs, removing duplicates
```

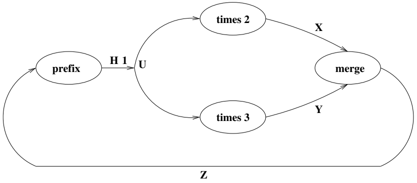
- Multiply each element of a list by a number:

```
% times(As, M, Bs): List Bs is obtained from number list As by
% multiplying each list element by M.
```

Example 5: Solving the Hamming problem via coroutining

- For this we add the block declaration
  ```
  :- block times(-, ?, ?).
  ```
  Meaning: suspend pred. `times` if the first arg. is an unbound variable
- Also, suspend pred. `merge` if the first or second arg is unbound
  ```
  :- block merge(-, ?, ?), merge(?, -, ?).
  ```

```
% U is the list of the first N (2,3)-Hamming numbers
hamming(N, U) :-
    U = [1|_], times(U, 2, X), times(U, 3, Y), merge(X, Y, Z),
    prefix_length([1|Z], U, N).     % A predicate from library(lists)
                  % prefix_length(L, P, N): L has a prefix P of length N
```



Part III – Constraint technology

Example 6: The 711 problem (David Gries, May 1982)

`https://www.cs.cornell.edu/gries/TechReports/82-493.pdf`

One day, a customer bought four items at a 711 store (a chain of stores in the US). The cashier bagged them and said:

- *That will be $7.11, please.*
- The customer asked: *Is it $7.11 because this is a 711 store?*
- *No*, replied the cashier, *I multiplied the prices together and got $7.11.*
- *But you are supposed to **add** them, not multiply them*, said the customer.
- *Oh, you're right!* exclaimed the cashier
- *Let me recalculate . . . that will be $7.11.*

Can you find out the price of each of the four items, based on the above conversation?

Note: calculations are assumed to be exact, no rounding!

We will use `library(clpfd)`: Constraint Logic Programming over Finite Domains